WESTMINSTER SCHOOL
THE CHALLENGE 2017

MATHEMATICS III

Wednesday 26 April 2017

Time allowed: 1 hour 30 minutes

You may not use a calculator for this paper.

All your working should be clearly shown.

You should attempt all the questions.

Please write in black or blue ink.

1 Ravi buys 24 cookies. Sam buys 29 cookies, and pays 20% more than Ravi.
Which of them got the better value? Show clearly how you decided.

2 When buying a lottery ticket costing £40, Alex pays £18, Ben pays £12 and Chris pays £10. When they win a big prize, they share the prize in the same ratio as the amounts they paid and Alex gets £3675 more than Ben. How much was the prize?

3 The ingredients for a large Madeira cake are:

400 grams of flour
250 grams of sugar
150 grams of butter
4 eggs.

Tom buys flour and sugar in 1 kg bags; he buys butter in 350 g packets and he buys eggs in boxes of 6.

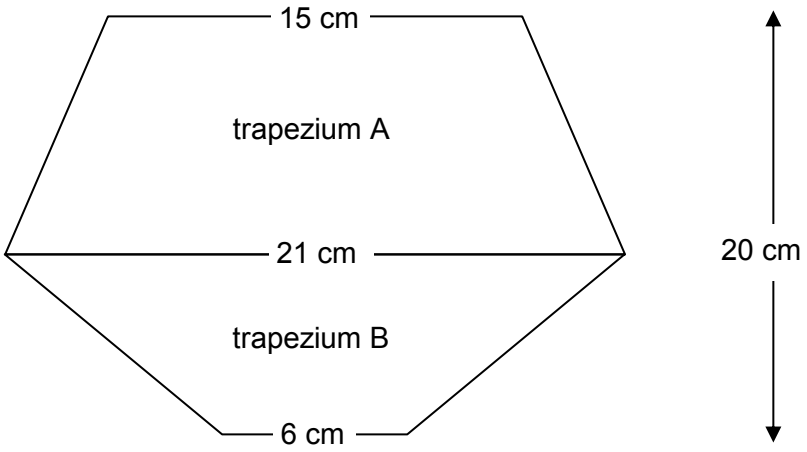
What is the smallest number of large Madeira cakes Tom can make if he is to use up a whole number of bags of flour, a whole number of bags of sugar, a whole number of packets of butter and a whole number of boxes of eggs?

4 Rob and Sam each goes for a walk. Sam walks twice as far as Rob, at one-and-a-half times the speed, and takes 40 minutes longer. How long did Rob walk for?

5 On his stall, Tom has is selling book bags and T-shirts. At the start of the day, he has 11 book bags and 13 T-shirts to sell.
In the morning, he charges £5 for a book bag, and £6 for a T-shirt. He takes in £65 in the morning.
In the afternoon, to try to clear his stock, he charges £6 for a book bag and £5 for a T-shirt. He takes £64 in the afternoon, and has no bags or shirts left.

Use an algebraic method to work out how many book bags he sold in the morning.

6 The diagram shows two trapezia with the same base.



The height of the whole figure is 20 cm and the area of trapezium A is twice the area of trapezium B. What is the height of each trapezium?

7 In the online role-player game Empire Quest, players amass gold coins and spend them in the marketplace on either spells, potions or maps. Each of these items costs a whole number of gold coins.

- Lucas buys 1 spell, 10 potions and 7 maps, spending 77 gold coins.
- Majid buys 1 spell, 3 potions and 4 maps, spending 41 gold goins.

Determine the price of each item.

8 **a** Multiply out and simplify $16(n+1)^2 + (n-1)^2$.

b By substituting $n = 10$ in your result from **a**, show how to write 2017 as the sum of two square numbers.

9 A *smooth* list of numbers is one where each number in the list (except the first and last) is the mean average of the two either side of it, so 7, 9, 11, 13 is a smooth list of numbers, because 9 is the average of 7 and 11, and 11 is the average of 9 and 13.

a Find the missing entries P and Q in this smooth list:
8·2, P, 11·6, Q.

b Find the missing entries R, S and T in this smooth list:
8·5, R, S, T, 21·5.

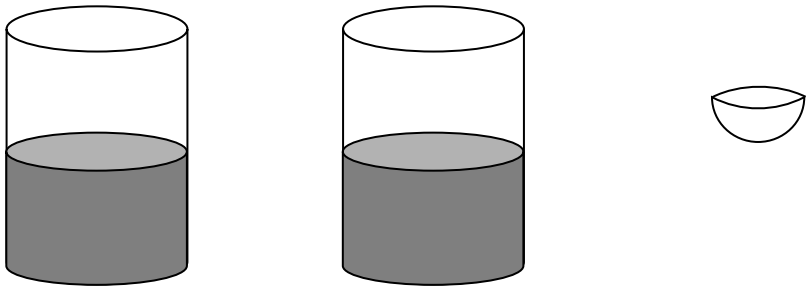
c Find, in terms of a and b, the missing entries U, V and W in this smooth list:
U, a, b, V, W.

d In a long smooth list, the 4th entry in the list is 11, and the 7th entry in the list is 20. Write a formula for the n^{th} entry in the list.

e In a long smooth list, the 1st entry in the list is a, and the 2nd entry in the list is b. Write a formula in terms of a, b and n for the n^{th} entry in the list.

10 **Reminder:** The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.

The diagram shows two identical cylindrical containers, with some water in, and a hemispherical scoop which is used to take water out of, or put water in to, the cylinders. The radius of the cylinders is $2r$, and the radius of the scoop is r.



When the cylinders are both half full, two scoops full of water are taken out of one cylinder, and five scoops full of water are added to the other cylinder. Then, the second cylinder contains twice as much water as the first.

Find the height of the cylinders in terms of r.

11 I have 70 litres of a mixture of 20% red paint, 35% purple paint and 45% blue paint. Without adding any blue paint, how much red and how much purple paint should I add so that I end up with a mixture of 25% red, 40% purple and 35% blue paint?

12 **a** Sam is trying to find the total of the whole numbers from 1 to 2017.
He imagines writing out a list of these numbers twice.

1	2	3	4	…..	2014	2015	2016	2017
2017	2016	2015	2014		4	3	2	1

i Explain why the total of all the numbers he imagines writing down is 2018×2017, and hence show that the sum of the whole numbers from 1 to 2017 is 2 035 153.

ii How many odd numbers are there between 1 and 2017, inclusive?

iii Use Sam's method to find the sum of all the odd numbers from 1 to 2017.

b The diagram shows how the numbers 1 to 11 can be written in a 6×6 square.

1	2	3	4	5	6
2	3	4	5	6	7
3	4	5	6	7	8
4	5	6	7	8	9
5	6	7	8	9	10
6	7	8	9	10	11

i By considering the copy of the square shown below, find the total of all the numbers in *one* square, using a method similar to Sam's. Explain your working clearly.

11	10	9	8	7	6
10	9	8	7	6	5
9	8	7	6	5	4
8	7	6	5	4	3
7	6	5	4	3	2
6	5	4	3	2	1

ii In an $n \times n$ square, completed in a similar way, what will be the largest number entered in the square?

iii What is the total of all the numbers in an $n \times n$ square, completed in a similar way?

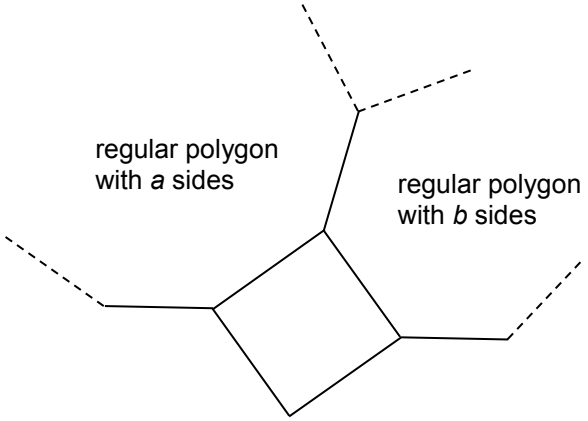
13 **a** Taddeo wants to find two positive whole numbers *a* and *b* which satisfy the equation $\frac{1}{a}+\frac{1}{b}=\frac{1}{4}$.

Taddeo notices that *a* and *b* could both be equal to 8, since $\frac{1}{8}+\frac{1}{8}=\frac{1}{4}$.

i He says "if the two numbers are not both equal to 8, then one of them must be less than 8". Explain how he could justify this claim.

ii Find all the pairs of values of *a* and *b* which satisfy Taddeo's equation.

b The diagram below shows two regular polygons and a square. One of the regular polygons has *a* sides and the other *b* sides.



i Explain why $\frac{360}{a}+\frac{360}{b}=90$.

ii How many sides could the two regular polygons have?

14 At lunch, the 38 students in a school are offered: a chicken sandwich, a slice of pizza or a vegetable wrap. On Monday and Tuesday one week the following statements are true.

- All 38 students have lunch each day.
- No student has the same lunch on both days.
- Altogether over the two days, 22 students have a slice of pizza.
- 13 students have a sandwich on Tuesday.
- 17 students have a wrap on Monday.
- 12 students have a slice of pizza on one day and a sandwich on the other.

a How many students had a sandwich on Monday?

b **i** Show that it is possible for all these statements to be true, if there was no student who had both pizza on Monday and a wrap on Tuesday.

ii What is the maximum number of students who could have had both pizza on Monday and a wrap on Tuesday?